On Study of Achievable Capacity with Hybrid Relay in Cognitive Radio Networks

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***Abstract*—This work studies the capacity limits of Cognitive Radio (CR) networks with a *hybrid relay* scheme in which PRimary (PR) nodes dominate the spectrum usage, while *coordinating* with secondary or CR nodes in forwarding packets. This is in contrast to previous efforts that are focused on the analysis with *cooperative relay* scheme where only secondary (CR) nodes participate in relay operation [1]–[3]. Intuitively, it is expected that the hybrid scheme can improve network capacity because primary nodes have higher power and transmission capacity compared to CR nodes and data transmission with hybrid relay scheme can be much faster and more reliable. Therefore, we aim to investigate such potential benefits with respect to cooperative relay. However, we find that given the total data rate W, number of usable channels $\|C\|$ and the number of CR nodes n in a network, the achievable capacity of CR networks with hybrid relay is of the order of $\Theta\left(\frac{W}{\|C\|n}\right)$. This result is much lower than the capacity of cooperative relay, which is of the same order as the capacity of multi-channel wireless networks (e.g., multi-channel multi-radio (MC-MR) networks), $\Theta\left(W\frac{1}{\sqrt{n\log n}}\right)$. It is plausible that PR nodes can potentially become bottlenecks when they are used as high-priority relay nodes in the hybrid scheme. Thus, our study suggests that for delay-sensitive applications such as real-time traffic, hybrid relay is a good option; otherwise, cooperative relay is preferred.**

I. Introduction

The demand for wireless spectrum has been growing rapidly and thus spectrum scarcity is becoming a severe problem. However, Federal Communications Commission (FCC) [4] has shown that traditional fixed spectrum allocation can be very inefficient, considering that most of the time, the assigned spectrum is not used. Thus, Cognitive Radio (CR) networks are considered as a promising paradigm to solve the problem of bandwidth limitation and inefficient spectrum utilization by dynamically exploiting spectrum [5]. In CR networks, unlicensed or secondary nodes (CR nodes) are allowed to opportunistically utilize spectrum bands assigned to licensed or PRimary (PR) nodes as long as they do not induce harmful interference to PR nodes. Here a "cognitive radio" is a radio that can change its transmitter parameters (operating spectrum, modulation, transmission power and communication technology) based on interaction with the environment in which it operates [4], [5]. Recently, there have been an explosion of interest in CR networks. New functions, such as spectrum sensing, spectrum mobility and spectrum sharing, and centralized and decentralized MAC protocols of CR networks have been studied in [3], [5]–[9], which have greatly advanced our understanding of the CR network architecture. Also the concept of CR has been embraced by the IEEE 802.22 Working Group that is working on standard for secondary use of the spectrum currently allocated to the television service [10].

Although CR networks have spurred great interest and excitements in both industry and academy, there is a lack of understanding of capacity limits in CR networks. It is important to be reminded that CR can be vastly more powerful and flexible than *multi-channel multi-radio* (MC-CR) technology that has been actively researched in recent years [11]–[13]. In MC-MR, *hardware-based* radio technology, where each radio can only operate on a single channel at a time, is used. Therefore, in MC-MR networks, the number of concurrent channels is limited by the number of radio interfaces and all nodes share a set of "common channels". In contrast, as a *software-defined* radio, CR is capable of reconfiguring RF and switching to newly-selected channels; thus, the number of channels that can be used by CR is much larger. In addition, based on its physical location, CR node may have a different set of "available" channels, that means channels available for one CR node may not be available to its neighbors. Moreover, considering the coexistence on the same spectral resource of both CR and PR nodes in CR networks, what possible relay schemes are and whether the cooperation between CR and PR nodes are beneficial to network capacity or performance remain unanswered. In CR networks, besides the traditional cooperative relay as in MC-MR, where only CR nodes participate in relay operation, PR nodes, which are dominant in channel usage, can coordinate relay among CR nodes. Hence hybrid relay scheme is possible. Therefore, although there have been many works on capacity of single-channel and MC-MR networks [12]–[15], considering the possible cooperation between CR and PR nodes and *shared common channel constraint*, which are unique in CR networks, these results may not be applicable in CR networks.

In this paper, we first investigate the possible hybrid relay in large-scale multi-channel CR networks, where PR nodes are dominant in channel usage, while coordinating relays among CR nodes. We find that the critical transmission range necessary for connectivity of CR networks under hybrid relay is different from that of MC-MR networks. Then based on this critical transmission range, we analyze the achievable capacity of multi-channel CR networks. The remainder of this paper is

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organized as follows. We formulate the problem and present a hybrid relay scheme in Section II. In Section III, we derive the capacity upper and lower bounds of CR networks using the hybrid relay scheme. We discuss the benefits of hybrid relay and conclude the paper in Section IV

II. Network Models and Hybrid Relay

A. System Models

In this paper, we study a cognitive radio network consisting of a set $V = \{v_1, v_2, \ldots, v_n\}$ of n CR nodes and a set $U = \{u_1, u_2, \ldots, u_m\}$ of m primary nodes in a unit square area $\Omega = \left[0,1\right]^2$ and $m = O(n)$. We assume that CR nodes can use a set $C = \{1, 2, \ldots, \|C\|\}$ of channels, each with data rate $\frac{W}{\|C\|}$, where $\|C\|$ denote the cardinality of C and the channels are numbered in the order of increasing frequency. Hence the total data rate is W. The following assumptions are used in this paper.

Node locations: The CR nodes are distributed according to Poisson point process (p.p.p) with density n in Ω and each CR node is equipped with a CR. PR nodes are distributed according to a Poisson point process (p.p.p) with density m over Ω and each PR is equipped a hardware-based radio (or multiple hardware-based radio (MR)). Among the m PR nodes, there exist L high-power high-bandwidth nodes (PR BSs e.g), which are called PRR nodes for convenience in this paper. These nodes form a PRR network and their positions are known to CR nodes. For example, the PRR nodes are placed regularly at positions $\left(\frac{1}{2\sqrt{L}} + i\frac{1}{\sqrt{L}}, \frac{1}{2\sqrt{L}} + j\frac{1}{\sqrt{L}}\right)$ with $0 \le i \le \sqrt{L} - 1$ and $0 \le j \le \sqrt{L} - 1$. Clearly, these L regularly distributed PRR nodes divide the region Ω into L subregions as Voronoi diagrams with side length $\frac{1}{\sqrt{L}}$ (see Fig. 1). Here we generally assume that L is finite and a square of some integer for simplicity, and use S_j denote the subregion centered at the primary node u_j.

Channel sets: Different primary nodes may have license to different subset of channels and the primary nodes sharing the common subset of channels form a primary network. We focus on the case where $\|C\| = \Theta(logm) = O(logn)$. This is justifiable since in large scale deployments, the number of channels will typically be much smaller than the number of nodes. Moreover, as shown in [1], when $\|C\| = O(logn)$, the capacity is independent of the number of interfaces. That is, we only need one interface to use the multiple channels. This will greatly simplify our design and analysis.

Interference model: In this paper, a protocol interference model [14] will be used because this model highlights the effects of transmission and interference range. Further, it has been shown that under some conditions, the more realistic Physical Model can be reduced to the Protocol Model [14]. Let r and R denote the transmission range of CR and hardware-based radio, respectively. CR node v_i can successfully transmit to CR v_j over channel ch_k if $\|v_i - v_j\| < r$ and $\|v_l - v_j\| \ge (1 + \Delta_1)r$ for each CR v_l transmitting over ch_k for some constant Δ_1. Similarly, hardware-based radio u_i can successfully transmit to hardware-based radio u_j over channel ch_k if $\|u_i - u_j\| < R$ and $\|u_l - u_j\| \ge (1 + \Delta_2)R$ for each hardware-based radio u_l transmitting over ch_k for some constant Δ_2.

B. Hybrid Relay VS. Cooperative Relay

Most of the existing works on CR networks assume that there is no inter-operation between PR and CR nodes, such as in [1]–[3]. PR nodes are oblivious to the presence of CR nodes and behave as if no CR activity was present. CR nodes, instead, sense the radio environment in search for "spectrum opportunity" (a time duration on a channel during which the channel can be used by CR nodes without interfering with the channel's PR nodes) and then exploit the detected "opportunities" dynamically. In this framework, CR networks use cooperative relay, in which only CR nodes participate in relay operation, to transmit. An example of this case is shown as the communication between v_1 and v_2 in Fig 1.

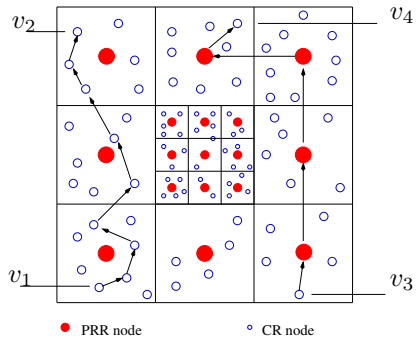


Fig. 1. Node positions and relay schemes. The perimeter is an extended version of the grids.

In this paper, we present a hybrid relay scheme, where high-power high-bandwidth PR nodes (PRR nodes) are dominant in channel usage, while coordinating relays among CR nodes. Particularly, as shown in Fig 1, when the source CR node and its destination CR node fall into the same subregion, the source CR node attempts to reach its receiver in multi-hop by using other CR nodes in the same subregion as relays. Otherwise, the source node will reach the closest PPR node first, and then the latter relay the data to the PRR node which is closest to the receiver (may use some other PRR nodes as relay nodes). At last, the PRR node closest to the destination node relays the data to the destination by one- or multi-hop. An example of this case is shown as the the communication between v_3 and v_4 in Fig 1.

Gupta and Kumar [14] have shown that, even all m PR nodes participate in relays, the capacity gain is bounded by $\sqrt{\frac{m}{n} + 1}$-fold, where n is the number of CR nodes. Thus hybrid relay scheme may not improve the capacity. However, comparing to the low-cost, low-power cognitive radio transceivers, the PRR nodes have higher power, higher bandwidth and higher reliability, thus data transmission in hybrid scheme is much faster and more reliable. Furthermore, for the PRR network, when the required traffic demand is satisfied, instead of increasing transmission rate, the PRR network is more interested

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benefit in other format, for example revenue. To coordinate relay among CR nodes, PRR nodes can improve their own revenue by charging CR nodes extra cost, which is proportional to data being relayed.

Note that a prerequisite in CR networks is that CR nodes cannot interfere with PR nodes. We justify our hybrid relay scheme by showing that with some additional functions, the PRR network does not have to change its protocol and the QoS of the PRR network would not be influenced due to participating in relay among CR nodes. Specifically, we assume that CR nodes communicate with PRR nodes using a control channel. And we further assume that the PRR node has infinite buffer and two queues. One queue is for the primary network data (primary queue) and the other is for CR network data (secondary queue). Primary queue has higher priority and the data in secondary queue will be transmitted until the primary queue is empty. In this way, the PRR node do not need to change its protocol to relay the CR network data and the transmission of primary network data would not be influenced.

III. Capacity Limits of CR Networks

As discussed earlier, because of the unique characteristics in CR networks, such as possible cooperation between CR and PR nodes and *shared common channel constraint*, the existing results on capacity limits in single channel or MC-MR networks [12]–[15] may not be applicable in CR networks. In this section, we formally derive the achievable capacity of multi-channel CR networks.

A. Effects of Hybrid Relay: Probability and Connectivity

1) Existence of Shared Channels: As mentioned in Section I, one of the main differences between MC-MR networks and CR networks is that in CR networks, there may not be a common set of channels among an arbitrary pair of CR nodes due to PR nodes in the vicinity and imperfect sensing. While this observation does reflect the fact in CR networks, it induces another problem, that is, even for two CR nodes within transmission range r, they can communicate with each other only when they share common channels. We show later that this requires larger transmission range of CR nodes to make the CR network connected. Here we characterize this difference by the probability, denoted by P_s, that any pair of CR nodes shares at least one common channel so that the communication between them is possible. This probability will be used in subsequent sections for deriving the capacity limits in CR networks.

We first show that at any point $z \in \Omega$, the usage efficiency of any channel is bounded by some constant $1-p<1$. Given $z \in \Omega$, the interfering area A_z is a circle center at z with radius $(1+\Delta_2)R$. That is, the channels used by PR nodes in this area can not be used by the CR node at point z. Let y_j be a indicator function such that $y_j = 1$ if the PR node u_j is in the interfering area and $y_j = 0$ otherwise. Since PR nodes are randomly and independently distributed, $P(y_j = 1) = \pi R^2(1+\Delta_2)^2$ for all j. Let y denote the number of PR nodes in the interfering area. Thus $y = \sum_{j=1}^{m} y_j$ and $E(y) = m \cdot \pi R^2(1+\Delta_2)^2$. Recall from [16], usually $R = \Theta(\sqrt{\frac{logm}{m}})$ and thus $E(y) = \Theta(logm)$. And by Chernoff-bound theorem (see [15], [17] for details), we can show that $|y - E(y)| \to 0$ as $m \to \infty$. Thus $y = \Theta(logm)$ with high probability. We assume that the number of PR nodes using any channel is of the same order. This assumption is justifiable because we do not consider the transmission quality difference among channels in our theoretical analysis. Thus, a randomly chosen PR node has the same probability to use any channel. That is, equivalently, for any channel $ch_k \in C$, it is used by a PR node in A_z with probability $\Theta(\frac{1}{\|C\|}) = \Theta(\frac{1}{logm})$. Thus ch_k is available at z with probability $(1-\Theta(\frac{1}{logm}))^y \approx e^{-\frac{y}{\Theta(logm)}} = e^{-\epsilon}$ for some constant $\epsilon > 0$ when m is sufficiently large. Let $p = e^{-\epsilon}$ and note that $p < 1$. This result corresponds the FCC measurement [4] which shows that 70% of the allocated spectrum in US is not used temporally and geographically.

For any CR node v_i, let A_i denote the subset of channels being sensed by v_i and $\|A_i\|$ be the cardinality of A_i. For simplicity, assume $\|A_i\| = \|A\|$ for all i. For any two CR nodes v_i and v_j, let $K_{i,j}$ denote the set of available channels shared by v_i and v_j. That is $K_{i,j} = \{ch_k : ch_k \in A_i \cap A_j$, and ch_k is available at v_i and v_j $\}$, and $P_s(k)$ denote the probability that channel k is shared by v_i and v_j. Then we have

$$P_s(k) = P(k \in K_{i,j}) = \Big(\frac{\|A\|}{\|C\|}\Big)^2 \times p^2$$

$$\begin{aligned} P_s = P(K_{i,j} \neq \emptyset) &= 1 - P(K_{i,j} = \emptyset) \\ &\approx 1 - \Big[1 - \Big(\frac{\|A\|}{\|C\|}\Big)^2 \times p^2\Big]^{\|C\|} \\ &\approx 1 - e^{-\frac{(\|A\|p)^2}{\|C\|}} \approx \frac{\|A\|^2 p^2}{\|C\|}. \quad (1) \end{aligned}$$

Note that in the above derivation, we assume that $ch_k \in K_{i,j}$ is independent of other channels. This approximation is reasonable when $\|C\|$ is sufficiently large.

2) Connectivity Constraint in CR Networks: The connectivity of CR networks is a prerequisite for communication since the throughput $\lambda(n) = 0$ if a CR network is disconnected. Because of the hybrid relay scheme and *shared common channel constraint*, the critical transmission range necessary for connectivity in CR networks may be different from that in MC-MR networks. First, when a hybrid relay scheme is used, the entire network is connected if and only all CR nodes and the PR node falling in each subregion S_i form a connected subnetwork. In this case, the transmission range of CR nodes can be smaller than a fully connected network of CR nodes. However, considering the *shared common channel constraint*, two CR nodes within transmission range maybe can not communicate with each other because of lack of "shared common channels". This requires larger transmission range for connectivity. Here we formally derive the necessary condition for connectivity of CR networks using a hybrid relay scheme.

Theorem 1: Given a CR network with n nodes uniformly deployed in a unit area Ω, each with transmission range r, if $P_s \pi r^2 = \frac{\log n + \kappa(n)}{n}$ and the network is connected asymptotically almost surely, then $\lim_{n\to\infty} \kappa(n) = \infty$.

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Proof: See Theorem 5 in [18] for details. ■

Theorem 1 gives a necessary condition of a multi-channel CR network to be connected using a traditional cooperative relay scheme, which implies $r > \sqrt{\frac{\log n}{P_s n}}$. Consider CR networks using the hybrid relay scheme and let N_i denote the number of nodes falling in subregion S_i, we have

Lemma 2: For $1 \leq i \leq L$, $N_i = \frac{\rho n}{L}$ for some constant ρ with high probability.

Proof: See Lemma 1 in [19] for details. ■

Since there are N_i CR nodes randomly and uniformly distributing on the subregion S_i with area $\frac{1}{L}$, each with transmission range r, by Theorem 1, we have

Lemma 3: If a CR network with hybrid relay is connected with high probability and $LP_s\pi r^2 = \frac{\log N_i + \kappa(N_i)}{N_i}$, then $\lim_{N_i \to \infty} \kappa(N_i) = \infty$.

Lemma 3 gives a necessary condition for a connected multi-channel CR network with a hybrid relay scheme, which implies the transmission range r must be at least $\Omega\Big(\sqrt{\frac{log(\frac{\rho n}{L})}{P_s \rho n}}\Big)$ for some constant ρ.

So far, we have investigated possible cooperation between CR and PR nodes and the *shared common channel* problem in CR networks. We find that the critical transmission range for connectivity of CR networks is different from that of MC-MR networks due to the *shared common channel constraint* and hybrid relay. Specifically, in hybrid relay, a necessary condition for connectivity is $r = \Omega\Big(\sqrt{\frac{log(\frac{\rho n}{L})}{P_s \rho n}}\Big)$, where $P_s \approx \frac{\|A\|^2 p^2}{\|C\|}$. Next, we will derive the achievable capacity in CR networks based on our analytical results.

B. Capacity Upper Bound of CR Networks

Throughout the paper, we consider *throughput* as the time average of the number of bits per second that can be transmitted by each CR node to its destination CR node, similar to [14]:

Definition 1: (Feasible throughput). The throughput of $\lambda(n)$ bits per second for each CR node is *feasible* if there exists a spatial and temporal scheme for scheduling transmissions, such that by operating the network in a multi-hop fashion and buffering at intermediate nodes (CR or PR) when awaiting transmission, every CR node can send $\lambda(n)$ bits per second on average to its chosen CR destination node. And We say that the capacity of a class of Random Networks is of order $\Theta(f(n))$ bits per second if there are deterministic constants $c_1 > 0$ and $c_2 < \infty$ such that

$$\lim_{n\to\infty} P(\lambda(n) = c_1 f(n) \text{ is feasible}) = 1,$$

$$\lim_{n\to\infty} P(\lambda(n) = c_2 f(n) \text{ is feasible}) < 1.$$

In this paper, we focus on the case when $\|C\| = O(\log n)$. As mentioned in [12], when $\|C\| = O(\log n)$, the capacity upper bound is dominated by the connectivity constraint. Specifically, given the number of hops taken by each packet $H(b)$ for packet b and the number of simultaneous transmissions on each channel S, the total number of transmissions required in time interval T is at least $\lambda(n)TnH(b)$. Then we have $\lambda(n)TnH(b) \leq TSW$. Since $S = O\big(\frac{1}{r^2}\big)$, we further have $\lambda(n) = O\Big(\frac{W}{r^2 nH(b)}\Big)$. Next we derive the lower bound of $H(b)$ to achieve the upper bound of capacity limit. Let Q denote the length transversed by bit b among CR nodes and $H(b) \geq \frac{Q}{r}$. By using Lemma 9 from [17] and Lemma 4 from [19], we derive $Q = \Theta(\frac{1}{\sqrt{L}})$ with high probability, where L is number of PRR nodes as defined in Section II-A, for the source and destination CR nodes are either in the same subregion, or in different subregions. Thus, $H(b) = \Theta(\frac{1}{r\sqrt{L}})$ and $\lambda(n) = O\Big(\frac{\sqrt{L}W}{rn}\Big)$. Hence we have the following result:

Theorem 4: For a multi-channel CR network with hybrid relay, there is a deterministic constant $c < \infty$, such that

$$\lim_{n\to\infty} P\Big(\lambda(n) = \frac{cW\sqrt{L\rho P_s}}{\sqrt{n\frac{\rho n}{L}}} \text{ is feasible}\Big) = 0. \qquad (2)$$

Unfortunately, we show later that this upper bound is not achievable. After analysis, we find that in hybrid relay, CR nodes in subregion S_i send data to PRR node u_i in S_i first when their destination nodes falling outside of S_i. Thus, if most of source nodes have destinations outside the subregion, the PRR nodes have much burden to relay data, thus become bottlenecks.

Lemma 5: In any subregion S_i, with high probability, there exist at least $\Theta\Big(\frac{n}{L}(1-\frac{1}{L})\Big)$ nodes that will send data to PRR node u_i.

Proof: See [19] for more details. ■

Note that the data rate for any PRR node to receive from adjacent CR nodes is $\frac{W}{\|C\|}$. Therefore, by Lemma 5, we have the following tighter upper bound:

Theorem 6: For the multi-channel CR networks using hybrid relay, the capacity $\lambda(n)$ is at most $\lambda(n) \leq \Theta\Big(\frac{W}{\|C\|\frac{n}{L}(1-\frac{1}{L})}\Big)$ with high probability.

C. Capacity Lower Bound of CR Networks

In this subsection, we show that the capacity upper bound derived in the previous subsection is sharp by exhibiting a scenario where we can achieve capacity with the same order of the capacity upper bound. Specifically, we provide a constructive hybrid relay scheme and prove that the capacity achieved by our scheme matches the asymptotic upper bound in the order of $\Theta\Big(\frac{W}{\|C\|n}\Big)$. This scheme is based on a good approximation of a minimum connected dominating set ($MCDS$, see [20] for details) and we call this scheme as $MCDS$*-based routing scheme* in this paper.

Definition 2: (Connected Dominating Set (CDS)). Given an undirected graph $G = (V, E)$, a dominating set (DS) of G is a subset $DS \subset V$: each node $u \in V$ is either in DS or adjacent to some $v \in DS$. Here v is also called the "dominator" of u. A connected dominating set CDS of G induces a connected subgraph of G. $u \in CDS$ is called an $MCDS$ node and an edge between two $MCDS$ nodes is an $MCDS$ edge.

Before we construct an approximation of $MCDS$, we will first show the following lemma:

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Lemma 7: In CR networks, given the $MCDS$, there exists a schedule solution by which all $MCDS$ nodes are able to transmit in every Γ time slots for some constant Γ, which guarantees that each $MCDS$ node can transmit at rate of $\Theta\left(\frac{W}{\|C\|}\right)$.

Proof: From [17], [20], we know that for a random network with a single channel, we can construct an approximation of $MCDS$ and each $MCDS$ node has at most a constant number of neighbors on the $MCDS$; and thus an $MCDS$ node can transmit at least once in every Γ time slots for some constant Γ. In our case, for each subset $V_k \subset V$ of all CR nodes which can use the available channel $ch_k \in C$, we construct a connected dominating set $MCDS_k$ of V_k using similar scheme to [17], [20]. And each node $v \in MCDS_k$ can only transmit with channel ch_k even it can also use other available channels. The union $\cup_{ch_k \in C} MCDS_k$ plus at most $\|C\|$ channel transition nodes (which can communicate with nodes from two different channels), is a $MCDS$ for our CR radio network. For this $MCDS$, each node can transmit once in every every Γ time slots with some channel $ch_k \in C$. Consequently, the per-node data transmission rate achieved by each node on the $MCDS$ is at least $\frac{W}{\|C\|\Gamma}$. Thus the result. ■

The details of *$MCDS$-based routing scheme* are as follows:

1) Partition the unit area into squarelets, each of area $\alpha(n)$. The transmission range r is set to $\sqrt{8\alpha(n)}$, thereby ensuring that any two nodes from two squarelets (sharing a common side) can communicate with each other directly. The partition of a specific subregion is shown in Fig 2.
2) Let $V_k \subset V$ denote the subset of all CR nodes which can use channel $ch_k \in C$. At each squarelet SQ_l, we choose a node $v_k^l \in V_k$ for all $ch_k \in C$. Assume that v_k^l transmits only with channel ch_k. Thus $\bigcup_{k \in C} \bigcup_l v_k^l$ plus at most $\|C\|$ "connectors" (channel transition nodes), automatically form a $MCDS$. An example of such choice of $MCDS$ nodes in a fixed subregion is shown in Fig 2.
3) For each source node v_i in subregion S_j and its destination node v_d, if v_d also falls into S_j, we find an $MCDS$ node in each of these squarelets that are crossed by line $v_i v_d$. We connect these $MCDS$ nodes in sequence to form a path, denoted as $P(v_i, v_d)$, and data are transmitted from v_i to v_d through $P(v_i, v_d)$ (for example, see communication between v_1 and v_2 in Fig. 2). If v_d falls outside S_j, we find an $MCDS$ node in each of these squarelets that are crossed by line v_i to PRR node u_j. We connect these $MCDS$ nodes in sequence to form a path, denoted as $P(v_i, u_j)$, and data are transmitted from v_i to u_j through $P(v_i, u_j)$ first (see communication between v_3 and PRR u_j in Fig. 2), and then u_j will transmit the data to the PRR node u_d in the same subregion of v_d, maybe use other PRR nodes as relays (see Fig. 1). Finally, u_d will relay data to v_d through path $P(u_d, v_d)$, which is constructed similar to $P(v_i, u_j)$.
4) When v_i (or v_d or both) is not an $MCDS$ node, v_i (or v_d or both) transmits to its dominator (say v_i' and v_d') in the same squarelet first (see part (a) of Fig. 3). Then v_i' transmit to v_d (maybe v_d') with the above method. When v_i and v_d use different channels, v_i first transmit data to a $MCDS$ node v_d'', which uses the same channel with v_i and exists in the squarelet containing v_d, through path $P(v_i, v_d'')$ constructed above. And then v_d'' relay data to v_d, maybe use a transition node v_d'. (see part (a) of Fig. 3).

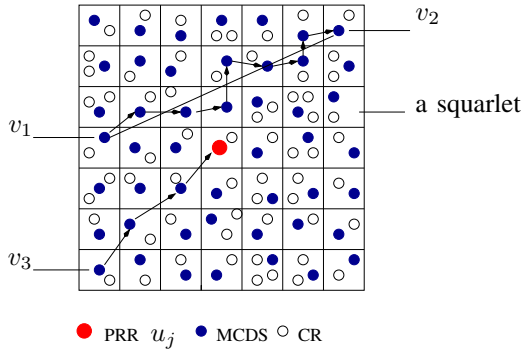
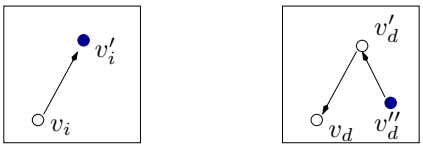


Fig. 2. *$MCDS$-based routing scheme*: communications in one specific subregion.



(a) (b)

Fig. 3. Possible communications in a fixed squrelet.

We show the validity of the *$MCDS$-based routing scheme* and the capacity achieved by this scheme in the following Lemmas. Let $\alpha(n) = \frac{100 \log n}{nP_s}$ and we have:

Lemma 8: In *$MCDS$-based Routing Scheme*, there exist at least $\frac{25 \log n}{P_s \|C\|}$ CR nodes in any squarelet, which can use the available channel k for each $k \in C$ with high probability.

Proof: See [18], [19] for details. ■

Lemma 8 ensures the correctness of *$MCDS$-based routing scheme*, which requires the existence of nodes being able to use channel ch_k for any $ch_k \in C$ at each squarelet. We can later show that the total data rate of all routing requests on each $MCDS$ node is at most $\Theta\left(\frac{W}{\|C\|}\right)$ almost surely based on our scheme, which guarantees that each CR node can transmit once every Γ time-slots by Lemma 7. This implies the total data rate achieved by all nodes is at least $n \cdot \frac{W}{\Gamma \|C\|}$. Therefore, if we can get the number N_c of nodes that get a copy of a data packet during its transmission from the source node to its destination, we can derive the lower bound throughput $\lambda(n) = \Omega\left(\frac{n \cdot \frac{W}{\Gamma \|C\|}}{N_c}\right)$.

Based on Lemma 4 and Lemma 5 in [19], we know that in hybrid relay scheme, the length between the source node v_i and v_d, denoted by Q is $\Theta(\frac{1}{\sqrt{L}})$ with high probability, where L is number of PRR nodes as defined in Section II-A. And the number of squarelets crossed by $v_i v_d$ is at most $\frac{\sqrt{2}Q}{\sqrt{\alpha(n)}}$. Thus

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the length L_p of the resulting path $P(v_i, v_d)$ satisfies

$$L_p \leq \frac{\sqrt{2}Q}{\sqrt{\alpha(n)}} \times \sqrt{2\alpha(n)} = \Theta(\frac{1}{\sqrt{L}}). \quad (3)$$

Let $A(P)$ be the region covered by all transmitting disks of all transmitting nodes in the path $P(v_i, v_d)$. Clearly, the area of $A(P)$, denoted by $|A(P)|$, is at most $|A(P)| \leq 2rL_p + \pi r^2 < \psi$ for some constant ψ when n is large enough.

Theorem 9: (the upper bound of the data copies) With the $MCDS$*-based routing scheme*, the number of nodes N_c that get a data copy is, with high probability, at most $N_c \leq \frac{3}{2}\psi n$, here ψ is a constant.

Proof: See Lemma 12 and Lemma 13 of [19] for details. ■

Based on Theorem 9, we have the following theorem.

Theorem 10: (the lower bound of the capacity) The capacity for multi-channel CR networks with hybrid scheme is at least $\lambda(n) = \Omega(\frac{W}{n\|C\|\Gamma})$.

Note that this lower bound matches the upper bound in Section III-B in the order of $\Theta(\frac{W}{n\|C\|})$, which implies $\lambda(n) = \Theta(\frac{W}{n\|C\|})$ is achievable in multi-channel CR networks with hybrid relay scheme. Also note that the correctness of Theorem 10 requires that the total data rate of all routing requests on each $MCDS$ node is no more than $\Theta(\frac{W}{\|C\|})$. Since the achieved capacity is $\lambda(n) = \Omega(\frac{W}{n})$, it is clear that the traffic load on any routing node ($MCDS$ node) satisfies this requirement.

IV. Discussion and Conclusions

As mentioned earlier, most of the existing works [1]–[3] have studied cooperative relay in CR networks, where only CR nodes participate in relay operation. For cooperative relay, the achievable capacity is of the same order as capacity of MC-MR networks $\Theta\Big(W\frac{1}{\sqrt{n\log n}}\Big)$, since the only difference is the probability of shared channels. Actually, this can be shown easily through similar techniques in [15].

For hybrid relay, we find that by making the best use of PRR nodes' higher capacity in power and bandwidth in comparison to CR nodes, data transmission can be much faster and more reliable. However, if there exist many CR nodes whose destinations fall into different subregions and these CR nodes may send routing requests to the PRR nodes nearby, *the PRR nodes can potentially become the routing hot spots and hence network bottlenecks.* Therefore, the capacity under hybrid relay scheme is much lower than that under cooperative scheme. For application purpose, we can categorize data packets into two groups. For the real time data which need to be delivered to the destinations within a short time, the hybrid relay scheme is a good option; otherwise cooperative relay is preferred for data packets without delay constraint, such as delay tolerant networks. Also, as mentioned in Section II-B, besides CR nodes, PRR nodes can also benefit from hybrid relay by charging CR nodes extra cost for relaying.

In this paper, we investigate some characteristics unique in CR networks, such as *shared common channel constraint* and possible cooperation between CR and PR nodes, and present and study a hybrid relay scheme. We investigate the effect of these characteristics on critical transmission range and achievable capacity. We derive the capacity limits of CR networks under hybrid relay and find that in contrast to our intuition, hybrid relay can not improve the network capacity. However, hybrid relay can provide faster and more reliable transmissions for important data packets. Furthermore, to help relay among CR nodes, PRR nodes can also improve their revenue by charing CR nodes extra cost, which is proportional to the data being relayed.

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